

Sets – Exercises

Exercises for Chapter 1 of Steinhart, E. (2009) *More Precisely: The Math You Need to Do Philosophy*. Broadview Press. Copyright (C) 2009 Eric Steinhart. Non-commercial educational use encouraged! All others uses prohibited. (Version 1)

1. Collections

Write out the following:

The set of A:

The set of the set of A:

The set of A and B:

The set of both A and the set of A:

The set of A, B, and C:

If x is $\{A, B\}$ and y is $\{C, D\}$ then write out:

$\{x\} =$

$\{x, y\} =$

$\{\{x\}\} =$

$\{\{x\}, y\} =$

Answer the following (true or false):

$1 = \{1\}?$

$\{1\} = \{\{1\}\}?$

$\{1, 1\} = \{1, \{1\}\}?$

$\{1, B, 2\} = \{2, 1, B\}?$

$\{A, A\} = \{A\}?$

$\{A, A\} = \{\{A\}\}?$

2. Membership

True or false:

Is $A \in \{A\}?$

Is $\{A\} \in \{\{A\}\}?$

Is $A \in \{\{A\}\}?$

Is $\{B\} \in \{\{A\}, \{B\}\}?$

Is $\{A, B\} \in \{A, B\}?$

Is $\{\} \in \{A\}?$

3. Set Builders

Using the set $Y = \{1, A, 2, B, 3, C\}$, write out the following sets:

$\{x \in Y \mid x \text{ is a letter}\} =$

$$\{x \in Y \mid x \text{ is a number}\} =$$

If $X = \{A, B, C, 1, 2, 3\}$, $Y = \{A, B, C\}$, and $Z = \{1, 2, 3\}$, then write:

$$\{x \in X \mid x \text{ is also in } Y\} =$$

$$\{x \in X \mid x \in Z\} =$$

4. Unions

Write out:

$$\{a, b\} \cup \{1\} =$$

$$\{a, b\} \cup \{c, d\} =$$

$$\{a\} \cup \{b\} \cup \{c\} =$$

$$\{\{A\}\} \cup \{\{B\}\} =$$

$$\{a\} \cup \{\{1\}\} =$$

$$\{A\} \cup \{\{A\}\} =$$

5. Intersections

$$\{a, b\} \cap \{a\} =$$

$$\{a, b\} \cap \{\{a\}\} =$$

$$\{1, 2, 3\} \cap \{a, b\} =$$

$$\{\{\}, A\} \cap \{A, \{\}\} =$$

$$\{\{\}, 2, 3\} \cap \{\{\}\} =$$

$$\{a, b, c\} \cap \{b, c, d\} =$$

6. Subsets

$\{A, B\}$ is a subset of $\{A, B, C\}$?

$\{A\}$ is a subset of $\{A, B\}$?

A is a subset of $\{A\}$?

$\{A\}$ is a subset of $\{\{A\}\}$?

$\{A\}$ is a subset of $\{A, \{A\}\}$?

$\{A, B\}$ is a subset of $\{A, B\}$?

Write the subsets of $\{1, 2\}$:

7. Rank

Assume that $A, B,$ and C are individuals on rank 0.

$$\text{rank of } \{A\} =$$

$$\text{rank of } \{\{\{B\}\}, \{C\}\} =$$

$$\text{rank of } \{A, \{A\}\} =$$

$$\text{rank of } \{\{A, B\}, \{C\}\} =$$

8. Power Sets

The power set of $\{1, A\}$ is:

The power set of $\{Q\}$ is:

The power set of $\{\{\}\}$ is:

Write the power set of the power set of $\{A\}$:

9. Some Transformations of Sets

Suppose $X = \{A, B, C\}$.

Write the set that results from replacing each $x \in X$ with $\{x\}$:

Suppose $X = \{\{\{A\}, \{B\}\}, \{\{C\}\}\}$.

Write the set that results from replacing each $x \in X$ with $\cup x$:

10. Diagramming Sets

Use names or dots for sets and an arrow from x to y iff x is a member of y .

Draw the diagram for $\{A, B\}$.

Draw the diagram for $\{A, \{A\}\}$

Draw the diagram for $\{\{A\}, \{B\}\}$.

Draw the diagram for $\{\{\}, \{\{\}\}\}$.

11. Sets and Selections

Fill in the table with 0s and 1s to express all selections. Write the selected set in the rightmost cell of each row.

$\{\}$	A	$\{A\}$	

Now write the set of all sets from the rightmost cell of each row:

12. Numbers as Sets

Using the idea that n is the set of all numbers less than n , write out:

0

1

2

3

4

13. Diagramming Numbers as Sets

Draw a diagram for each of the numbers in exercise 12 above:

0

1

2

3

4

14. Iteration versus Accumulation

An *iterative hierarchy* says that every next level is just the power set of the previous level.

Consider this iterative hierarchy:

$$H(0) = \{A\}; \quad H(n+1) = \text{pow } H(n).$$

Write out levels $H(0)$, $H(1)$, and $H(2)$ of this iterative hierarchy:

$$H(0) =$$

$$H(1) =$$

$$H(2) =$$

A *cumulative hierarchy* says that every next level is the power set of the previous level unioned with the previous level.

Consider this cumulative hierarchy:

$$K(0) = \{A\}; \quad K(n+1) = \text{pow } K(n) \cup K(n).$$

Write out levels $K(0)$, $K(1)$, and $K(2)$ of this cumulative hierarchy:

$$K(0) =$$

$$K(1) =$$

Write out $\text{pow } K(1)$. (How can exercise 11 help you?)

$$K(2) =$$

Give an example of an object that appears on $K(1)$ but not on $H(1)$:

List all objects that appear on $K(2)$ but not on $H(2)$:

Explain why $K(n+1)$ is richer than $H(n+1)$ for $n > 0$:

15. Ordered Pairs

Diagram (Sue, Bob)

Diagram (Bob, Bob)

Diagram (Sue, {Sue})

Diagram ({}, {{}})

16. Cartesian Products

Write the Cartesian Product $\{A, B\} \times \{1, 2\}$.

Write the Cartesian Product $\{Abe, Bob, Sue\} \times \{Happy, Sad\}$.