On the plurality of gods

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Abstract: Ordinal polytheism is motivated by the cosmological and design arguments. It is also motivated by Leibnizian–Lewisian modal realism. Just as there are many universes, so there are many gods. Gods are necessary concrete grounds of universes. The god-universe relation is one-to-one. Ordinal polytheism argues for a hierarchy of ranks of ever more perfect gods, one rank for every ordinal number. Since there are no maximally perfect gods, ordinal polytheism avoids many of the familiar problems of monotheism. It links theology with counterpart theory, mathematics and computer science. And it entails that the system of universes has an attractive axiological structure.

Introduction

Ordinal polytheism argues for an infinite plurality of gods (Steinhart 2012). Unlike earlier polytheisms, ordinal polytheism does not posit more than one god at our universe. On the contrary, it associates our universe with exactly one god. This is *our local god* (or, to use the term *actual* indexically, it is the actual god). But ordinal polytheists will argue that our local god depends for its existence and its nature on prior gods. And they will argue that our local god has a counterpart at every possible concrete universe.

Ordinal polytheism begins with data manifest naturally to our senses; from those data, it reasons naturally to its theology, which is no more than a *likely story* about the gods (Plato, *Timaeus*, 29c–d).[1] Ordinal polytheism approaches its gods scientifically. They fall within the scope of the *formal sciences*. Ordinal polytheism is serious about the use of logic, mathematics, and computer science to analyse divine natures. Perhaps the most shocking aspect of ordinal polytheism is not that it posits many gods, but that its gods lack *mystery*. Gods, like minds and computers and sets, are fully open to rational enquiry.

Ordinal polytheism affirms that every god has degrees of power, intelligence, and benevolence. If those qualities make *perfection*, then every god has some degree of perfection. Yet the concept of *unsurpassable* perfection is empty (Steinhart (2012), sec. 4). The paradoxes of unsurpassability are well known (e.g. those associated with omniscience and omnipotence). However, ordinal polytheists also appeal to mathematics to justify the vacuity of unsurpassability: there are no unsurpassable ordinals, sets, computers, languages, games, or strategies (Steinhart 2003).[2] Those failures of unsurpassability strongly suggest that there are no unsurpassable minds and, plausibly, no unsurpassable gods.

Ordinal polytheism argues for an unsurpassably ramified tree of surpassably perfect gods. By dealing only with surpassable perfection, and by analysing it using the tools of the formal sciences, ordinal polytheism offers precise accounts of divine design and creativity; it also offers such accounts of divine action, simplicity, and infinity. Its gods fall into a divine hierarchy with the structure of the ordinal number line. They design and create a system of universes with an attractive axiological structure. And they avoid at least the logical problem of evil. Of course, our local god is one of these surpassable gods. And ordinal polytheism starts by arguing for the existence of our local god.

Two arguments for our local god

According to monotheists, our universe depends on some divine ground of physicality. This divine ground is our local god (*our god*). Ordinal polytheists agree that our universe depends on our god. Since ordinal polytheists seek scientific understandings of the divine, they seek evidence-based arguments for our god. One such type of argument for our god is the cosmological argument. Aquinas famously gave three versions of that argument (the first three ways in the *Summa Theologica*, Part 1, Q. 2, Art. 3). More recent versions have been developed by Leibniz (1697/1988), Meyer (1987), and Davis (1992).

Although there are many versions of the *Cosmological Argument*, they typically share a common form that looks something like this: (1) Some objects depend on other objects. (2) There are many descending dependency chains (in which x_0 depends on x_1, x_1 depends on x_2, and so on). (3) Dependency chains have no loops of any length (not even length 0). (4) Dependency chains cannot be infinitely descending.[3] (5) Thus, after finitely many steps, every dependency chain bottoms out in some independent object. (6) All dependency chains starting in the same universe bottom out in the same independent object. (7) Hence there exists one independent object on which all others in our universe depend: it is the necessarily existing ground of our universe. (8) As such, it is our god.

As the unique ground of our universe, our god is the continuously creative source of all things in our universe. Our god creates our universe much as a

minstrel makes music (Geach (1967), 60). The success of science at our universe justifies the thesis that our god plays according to a programme or score. By playing, the minstrel causes various notes to exist. The minstrel generates three *physical* relations among these notes: the temporal order among them; the spatial relations between high and low notes; and the causal relations within the series of notes. Following this analogy, the creative activity of our god generates the physical spatial, temporal, and causal relations among things in our universe.

Our universe contains all and only the physical things created by our god. To continue Geach's musical analogy, any notes played by the same minstrel are in the same universe. On the assumption that creation requires some sort of causal generation of the created object, and that anything that causally generates a concrete thing is itself concrete, ordinal polytheists say that our god is concrete. Contra Cameron (2009), our god is not an abstract object like a universal or a pure set. Yet here concreteness is a more general category than physicality: some concrete things (like our god) are deeper than physical.

Apart from the cosmological argument, another type of evidence-based argument for our god is the design argument. The *Organic Design Arguments* for our god run roughly as follows:[4] (1) Organisms are highly complex. (2) If something is as complex as an organism, then it requires an explanation. (3) The best explanation for this organic complexity is that there is some intelligent designer for organisms. (4) This designer is our god. However, ordinal polytheists do not accept this argument. Against design arguments at the organic level, ordinal polytheists say that evolution by natural selection is the best explanation for biological complexity. They affirm that: (5) The best explanation for the existence of complex organisms is that they have evolved; they exist on 'graded ramps of slowly increasing complexity' (Dawkins (2008), 139). Call such ramps *escalators*. (6) So, by inference to the best explanation, every organism exists on some *escalator*.

As a result of this dialectic, ordinal polytheists run the expected *Design Argument* at the cosmic level:[5] (1) Our universe is highly complex. (2) If something is as complex as our universe, then it requires an explanation. (3) As Geach says, universes are analogous to songs; but the best explanation for some song is not that it has been designed by some prior song; on the contrary, the best explanation is that it has been designed by some composer. (4) Accordingly, by inference to the best explanation, our universe has been designed by some cosmic composer. Following the monotheists, ordinal polytheists identify this cosmic composer with our god. To continue Geach's musical analogy, our minstrel is a composer as well as a player. The Cosmological and Design Arguments justify the thesis that our god is an intelligent designer-creator. Say a mind is *pure* iff it is concrete but deeper than physical. On this definition, our god is a pure mind.

The nature of the mind of our local god

Since our god is a mind, some account of mentality is needed. Although it is not without problems, *functionalism* remains one of the most widely accepted ways of understanding the mind. Alston (1985) applies functionalism to the divine mind and Lewis (1986, 106) says that it applies to 'a man or a beast or a god'.

But functionalist accounts are computational. Doore (1980, 154, fn. 1) tell us that 'if one were a functionalist of some type, perhaps one might argue that a mind is in many ways like a computer; therefore, the divine mind is probably in many ways like a computer'. Leibniz (1710/1996), Royce (1899/1927; 1901/1976), and Lodzinski (1998) portray our god as computing. And Leslie (2001; 2007) makes extensive use of the notion that divine minds are analogous to computers. Since our best scientific theories of mentality are computational, ordinal polytheists accept that our god is *analogous* to a computer. Of course, there are infinite computers vastly more powerful than classical Turing machines.[6]

Granted functionalism, ordinal polytheists are interested in computational accounts of divine design and creation. The *Leibnizian Account* is admirably clear. According to that Account, our god runs a *Leibnizian Search* (see Wilkins (2012), 34–35). The Leibnizian Search involves three steps. First, our god begins with some set of mental descriptions of universes (Leibniz (1710), secs. 414–417). Second, our god sorts those descriptions according to an axiological criterion. For one such sort, see Leibniz (1710, sec. 225); for another, see Rescher (1991, 171–172, 188, 191). The sorting algorithm arranges the cosmic descriptions into a series of axiological equivalence classes. On any Leibnizian Search, this series contains exactly one top axiological class. Third, our god somehow selects exactly one cosmic description from this top class. This is the best of all possible cosmic descriptions. Our god then actualizes that description – thus our god creates our universe.

On the Leibnizian Account, our god processes a considerable amount of information. Now, our best current science uses information to define *complexity*.[7] And while ordinal polytheists need not endorse any specific informational definition of complexity, they agree that the complexity of any thing is somehow proportional to the amount of information needed to define it. Informational definitions of complexity differ sharply from the compositional definitions traditionally used in philosophy of religion. Informational definitions do *not* say that an object is complex iff it has parts (nor that it is simple iff it lacks parts). They do *not* say that an object is complex iff it is distinct from its properties (nor that it is simple iff it is identical with its properties).

Since any definition of any possible object involves some information, every possible object has some positive degree of complexity. Ordinal polytheists therefore say that to be simple is to have some relatively minimal positive degree of complexity. For any type, an instance of that type is *simple* iff its complexity is

minimal among the instances of that type; it is *intricate* otherwise.[8] For instance, a universe is simple iff its complexity is minimal among the class of possible universes; it is intricate otherwise. Ordinal polytheists say that a god is simple iff its complexity is minimal among the class of possible gods; a god is intricate otherwise. Obviously, this account of divine simplicity differs sharply from the account offered by most monotheists. Because it is based on information (rather than on composition), this account allows simple gods to have parts and to be distinct from their properties. The task is now to examine the complexity of our god.

Granted that complexity is to be defined in terms of information, the Leibnizian Account entails that our god is extremely complex (Leslie (2001), ch. 1; Dawkins (2008), 178–179). To see this, start with cosmic descriptions (they are *descriptions* of universes; they are not universes). Any description of a universe is a fundamental physical theory along with any needed boundary conditions and settings of free parameters. The theory is expressed in some exact language. If this is right, then the complexity of a universe is proportional to the complexity of its fundamental theory. Many ways have been offered to measure theory-complexity (e.g. by counting quantifiers).[9] On all these ways, our universe is extremely complex. Any set that contains merely all the physically possible variants of our universe is enormous. On any adequate account of cosmic value, the axiological criteria used to rank cosmic descriptions are extremely subtle. Any ranking algorithm that uses such criteria is an extremely complex algorithm. Our god is therefore a mind that can run an extremely complex algorithm on an extremely large set of extremely complex mental representations. Our god is an extremely extremely extremely complex mind.

For ordinal polytheists, our god is the least complex pure mind able to design and create our universe.[10] The *infinitary thesis* states that since our universe is infinitely complex, our god must also be infinitely complex. If our universe has the kinds of complexity described by writers like Leibniz or Royce, then to design it our god must perform uncountably infinitely many operations on uncountably infinitely many bits.[11] However, if our universe is Democritean in the sense defined by Quine (1969, 147–152), and if it contains only finitely many material particles, then our god may only need to perform countably many operations on countably many bits. The *finitary thesis* states that since our universe is finitely complex, our god may be only finitely complex. Obviously, this thesis assumes the finite complexity of our universe. To that end, Lloyd (2002) argues that our universe is a process running on a quantum computer that has performed at most 10^{120} operations on at most 10^{90} bits. If that is right, then perhaps our god is merely finitely complex. And yet even if our god is merely finitely complex, it is not likely to be a simple god.[12]

The idea that our god is the concrete ground of our universe now meshes nicely with the idea that our god is analogous to a computer. Together these ideas inspire

the thesis that *our god is to our universe as hardware to software*. Our god supports our universe like a computer supports some software process. If this thesis is correct, then all changes in our universe supervene on divine actions; all the laws of our universe supervene on divine cosmic programming. This thesis refines Geach's idea that our god is to our universe as a minstrel is to his music. It has clear parallels with the field-theoretic interpretation of Spinoza (Bennett (1984); Viljanen (2007)). Perhaps the things in our universe supervene on our god much as the gliders in the game of life supervene on the computer running it (Dennett (1991)). Of course, to generate our universe, our god may need to perform continuous computation on real numbers (Moore (1996); Blum *et al.* (1998)).

Although informational accounts of complexity allow partless things to be complex, ordinal polytheists give three arguments that our god has parts. The first goes like this: our god is analogous to a computer; but all known computers have parts; so it is likely that our god has parts. The second quickly follows: our god is a mind; but all known minds have parts (Hume (1779/1990), 70–71); so once more it is likely that our god has parts.[13] The third runs this way: our god is a concrete thing that supports a universe with a part–whole structure. If any concrete thing supports such a universe, that thing must itself have some part–whole structure. Therefore, our god has some part–whole structure. Since the part–whole structure of our universe seems to bottom out in ultimate physical parts, it seems plausible to say that the part–whole structure of our god bottoms out in ultimate divine parts. And the ultimate parts of our god are complex by having many states interrelated in many ways.

On the basis of these considerations, ordinal polytheists say that our god has at least two kinds of parts. The first kind of part is temporal. Just as the temporality of some computer (or minstrel) generates the temporality of its programmatic processes (or songs), so the temporality of our god generates the temporality of our universe. Since software objects are generated stage by stage (like the frames of a movie), *stage-theory* seems to be the most appropriate theory of persistence for those objects (Sider (2001)). And since the hardware objects on which they supervene can also be described as persisting by having temporal stages, ordinal polytheists adopt stage-theory as the metaphysics of divine persistence. Granted stage-theory, our god is composed of temporal stages.

The second kind of part is functional. More precisely, each temporal stage of our god divides into its functional parts. Ordinal polytheists allow that some version of *homuncular functionalism* applies to our god (Dennett (1981), ch. 5; Lycan (1987), ch. 4). Some ordinal polytheists may even want to argue that, since it acts as a designer, our god has some language of thought (contra Kortum (2004)). However that may be, our god has two main functional parts: its *kernel* and its *shell*. The shell is the sum of all and only those parts of our god which participate in divine spatial relations. Such parts are needed to support the physical spatial relations among things in our universe. Hence the shell is that part of our god that

supports our universe. Any part not in the shell is in the kernel. And some parts must be in the kernel and not the shell, since the kernel runs at least the Leibnizian Search for the best universe to run on the shell. This division of our god into kernel and shell suggests that ordinal polytheism has affinities with *panentheism*.

Since anything that is involved in spatio-temporal causal relations is concrete, our god is concrete. Yet concreteness divides: while software objects are physical, the hardware objects on which they supervene are deeper than physical. As a hardware object, our god is deeper than physical. At our god, physics is the study of the patterns in and among the software objects that supervene on its shell. Since software objects have their own natures, which can be described without reference to either the hardware which realizes them or to the ways in which they are realized, the distinction between software and hardware is significant. The study of the software objects in the game of life refers neither to the computers which run the instances of that game nor to the ways in which they run them.

The progressive account of complexity

At this point, ordinal polytheism appears monotheistic. However, to motivate the existence of many gods, ordinal polytheists embrace the *Extended Design Argument:* (1) Our god is extremely complex. (2) But the Design Argument entails that if anything is as complex as our god, then it needs an explanation (e.g. Hume (1779/1990), 72–73; Doore (1980), 153–155). (3) Therefore, our god needs an explanation. (4) Since no thing explains itself, the explanation for our god lies in some other thing. Perhaps our god is explained by some prior god; yet that god is explained by some prior prior god (Dawkins (2008), 136, 146, 188). Perhaps our god is the end product of some 'version of Darwinism in another universe' (*ibid.*, 186). Or perhaps there is some better explanation.

Of course, it is hardly surprising to see Dawkins advocate an evolutionary account of divine complexity. But can such an account really be applied to our god? To answer this question, ordinal polytheists need to look at evolution more closely. For Dawkins (1996) and Dennett (1995, ch. 3), evolution can be described in computational terms. Accordingly, the *Library of Mendel* is an abstract multi-dimensional hyper-space whose books are possible genotypes. There is some information-based *complexity measure* that maps each genotype in the Library onto its complexity. Ordinal polytheists need not choose any detailed complexity measure now.[14] The complexity measure raises each genotype to its height in a *complexity landscape*. The *topography* of the landscape is familiar from earth: the least complex genotypes lie on the vast plains of simplicity; the more complex genotypes rest on higher ground. But the amount of land available at greater heights is always smaller, and there are very few genotypes on the peaks of great complexity.

An algorithm for walking through the Library is a function f that maps genotypes onto genotypes. Given a genotype as input, it produces another as output. A *walk* through the Library is a series of genotypes that starts with some genotype g and moves as directed by the repeated self-application of some algorithm f to its own output. If the application of f to its own output n times is denoted f^n, then every walk has the form $\langle g, f^1(g), f^2(g), \ldots f^n(g) \rangle$. Note that n may be finite or infinite. A walk is *cumulative* iff at least some of the complexity that exists at its end is complexity that has been acquired by the action of the algorithm itself over the course of its walk (thus exceeding any complexity that was given at the start). A cumulative walk *earns* some complexity over and above that which it inherits. Much may be squandered – yet some must be gained. An algorithm is cumulative iff all its walks are cumulative. Since at least some genotypes define organisms, at least some walks through the Library define series of organisms. It is therefore natural to extend the scope of f. A series of organisms is defined by f iff the genotype of each next organism is defined by the action of f on the genotype of the previous organism.

Ordinal polytheists accept the evolutionary thesis that all organic complexity on earth has been generated by cumulative algorithmic action. To justify that thesis, they offer the *Argument to Organic Ascent*. It goes like this: (1) The *Principle of Organic Ascent* says that for any complex organism O there exists some simple organism o and some cumulative algorithm f such that O is produced by the iteration of f on o. The cumulative algorithm f is evolution by natural selection. (2) The best explanation for all organic complexity is the Principle of Organic Ascent. (3) So, by inference to the best explanation, that Principle is justified. And it entails that for every complex organism O, there exists some simple organism o, some cumulative algorithm f, and some number n, such that O is $f^n(o)$. Every complex organism is at the end of a cumulative walk $\langle o, f^1(o), f^2(o), \ldots f^n(o) \rangle$. The Principle of Organic Ascent does *not* imply organic progress. It is consistent with massive waste of organic complexity; it permits our entire earthly ecosystem to crash down to simple organisms. It merely implies that any complexity squandered is ultimately complexity that was earned by cumulative algorithmic action.

The principle of ascent

Although the Principle of Organic Ascent is restricted to earthly biology, it can easily be generalized. On the basis of work by Chaisson (2001; 2006), ordinal polytheists argue for a version of that Principle which covers all the complex material things in our universe. On the basis of work by Dawkins (1996) and Dennett (1995), they argue for a version of that Principle which covers all possible complex things. The arguments that justify this fully generalized version are highly technical arguments based on the mathematics of complexity.[15] It is unfortunate that they cannot be presented here. Granted that the Principle of

Organic Ascent can be fully generalized, the result is the unqualified *Principle of Ascent*: for any possible complex thing X, there exists some possible simple thing x and some cumulative algorithm f such that X is produced by the iteration of f on x.

Ordinal polytheists accept the Principle of Ascent. As expected, sceptics will raise an *Objection from Implausibility*, which says that the Principle is far too speculative. Why should anybody believe it? To this question, ordinal polytheists have two answers: The first is that the mathematical reasoning that justifies the Principle of Ascent is independent of all contingencies, and thus covers all possible complex things.[16] The second is that, whatever flaws the Principle of Ascent may have, *it is the core of our best scientific explanation of complexity.* And ordinal polytheism aims to treat gods scientifically. Assuming that the Principle of Ascent is acceptable, ordinal polytheists turn to its entailments. It entails that for every possible complex thing X, there exists some simple thing x, some cumulative algorithm f, and some n, such that X is $f^n(x)$. Every possible complex thing is at the end of some cumulative walk of the form $\langle x, f^1(x), f^2(x), \ldots f^n(x)\rangle$. And it entails Dennett's *Principle of Accumulation of Design*, which states that the natures of more complex things are largely copied from those of simpler things (Dennett (1995), 72).[17]

On the basis of the scientific theory of complexity, ordinal polytheists offer the *Argument for Divine Ascent*. Some of this argument was already sketched by Dawkins (2008, ch. 4).[18] It goes like this: (1) There is some complexity landscape of divine natures and there are walks through this landscape. (2) Since it covers all possible complex things, the Principle of Ascent applies to complex gods. (3) Applied to such gods, it says that for any complex god G, there exists some simple god g and some cumulative algorithm f such that G is produced by the iteration of f on g. (4) Since the Principle is the best explanation for all possible complexity, it follows that, for every complex god G, there does exist some simple god g, some cumulative algorithm f, and some number n, such that G is $f^n(g)$. Every complex god is at the end of a cumulative series $\langle g, f^1(g), f^2(g), \ldots f^n(g)\rangle$. (5) Since our god is complex, it is at the end of some cumulative series of gods.

The Argument for Divine Ascent suggests that gods are produced by something like evolution by natural selection. This leads to the *Objection from Divine Solitude:* Since gods are the deepest concrete things, they are not parts of any deeper concrete system in which they could breed or compete; hence they cannot evolve by natural selection. Ordinal polytheists agree with monotheists that divine creation is *ex nihilo:* every god creates out of its own being without the involvement of any other things. Hence gods are solitary creators. Within any series of increasingly complex gods $\langle g, f^1(g), f^2(g), \ldots f^n(g)\rangle$, each next god is directly created *by and only by* its previous god. Since the Principle of Accumulation of Design applies within any such series, it follows that design accumulates because every god makes a more complex version of itself. For each god, f is not some

externality, but is internal to the complexity of that god. Hence the complexity of each next god is fully explained by the self-application of the complexity of its previous god.

The algorithm for divine ascent

Since divine ascent is constrained by divine solitude, each previous god produces the next god by something *analogous* to asexual reproduction. A god *self-reproduces* iff it designs and creates some more complex version of itself. Fortunately, there is no need crudely to picture gods as budding like yeast or hydra: gods are 'mind-producing minds' (Doore (1980), 154). Thus divine self-reproduction is deeper than physical.[19]

Assuming that gods self-reproduce, and that their self-reproduction is governed by the Principle of Ascent, then our god is at the end of *our lineage* of gods. Our lineage has the form $\langle G(0), G(1), G(2), \ldots G(k) \rangle$ where $G(k)$ is our god. Here k may be finite or infinite. Since our god exists necessarily, every god on which it depends also exists necessarily; hence all the gods in our lineage are necessary beings.[20] Our lineage is well-ordered and begins with *our simple initial god* $G(0)$. Our simple initial god is an ultimate necessary being. Its simplicity means only that $G(0)$ is such that no less complex god is possible. But how complex? If our initial god is not complex enough to self-reproduce, then the lineage of gods does not exist; but ordinal polytheists say it does exist; hence they say that our initial god is the least complex god that is able to self-reproduce.

The Argument for Divine Ascent entails that the later gods in our lineage are more complex than the earlier gods. On the computational analysis, complexity involves both *intelligence* and *power*. The Argument for Divine Ascent clearly requires that every god in our lineage has some power and intelligence. Otherwise, it would not self-reproduce, and our lineage would not exist. And that Argument also entails that from $G(0)$ to $G(k)$, the power and intelligence of gods increase. To understand how those divine qualities increase, ordinal polytheists turn to the analysis of god-making algorithms. An algorithm f is *theogenic* iff there is some simple god g and some n such that $f^n(g)$ is our god. Of course, theogenic algorithms are cumulative – but they also have other intriguing features.

Ordinal polytheists now offer an *Argument from Contrastive Probabilities* to the thesis that every theogenic algorithm always increases both power and intelligence together. It goes as follows: (1) A theogenic algorithm is *strong* iff it always increases both power and intelligence together. (2) Just as mountaineers who scale great peaks using both arms *are more common than* those who scale such peaks using only one, so theogenic algorithms that are strong are more common than those that are not. (3) The most likely explanation for our god is that it has been created by the iteration of the most common type of theogenic algorithm.

(4) So, the most likely explanation for our god is that it has been created by the iteration of some strong theogenic algorithm. (5) But the most likely explanation is the best. (6) So, by inference to the best explanation, our god has been produced by the iteration of some strong theogenic algorithm.

Although every god has some power and intelligence, those qualities alone do not suffice to generate our lineage. Those qualities define what every god *can* do. However, they do not define what any god *will* do. They do not supply any god with any *motivation* to do anything. Specifically, they do not supply any god with the desire to self-reproduce; yet if any god does not have such desire, then it does not self-reproduce, and our lineage does not exist. Therefore, every god in our lineage has some desire to self-reproduce. Since power and intelligence are positive qualities, and since any desire to produce more positivity is itself positive, any divine desire to self-reproduce is a positive desire. Any positive desire is some *benevolence*. So, every god in our lineage has some benevolence.

More precisely, the benevolence of any agent is its desire to do the best it can. And while this desire may vary in its intensity from agent to agent, it has a maximum. Maximum benevolence is the desire of any agent to do *exactly* the best it can. This is *optimality*. One way to clarify this optimality is to say that an agent is optimal iff it wants to do exactly what it ought to do and it ought to do the best it can do.[21] For any act P, if any optimal agent ought to do P, then it can do P and it wants to do P; but if it can and wants to do P, then it does P; so, if any optimal agent ought to do P, then it does P. Every god is associated with some set of obligations; simpler gods have smaller sets of simpler obligations.

Ordinal polytheists now adapt the logic of the Argument from Contrastive Probabilities to justify several points about theogenic algorithms. A theogenic algorithm is *well-equipped* iff it starts with an optimal simple god and it is *careful* iff it always preserves optimality. A theogenic algorithm is *athletic* iff it is strong, well-equipped, and careful. Once more theogenic algorithms are analogous to mountaineers who scale great peaks: by far the most common theogenic algorithms are athletic. Granted the logic of the Argument from Contrastive Probabilities, ordinal polytheists say that the best explanation for our god is that it has been produced by the iteration of some athletic theogenic algorithm.

Our god is produced by the iteration of some athletic theogenic algorithm f. If this is correct, then f always increases both power and intelligence and it always preserves optimality. For the sake of convenience, ordinal polytheists introduce the term *perfection* to capture the way f combines these qualities. God x is *more perfect than* god y iff x is more powerful and more intelligent than y and both x and y are optimal. Since there are variable degrees of power and intelligence, there are variable degrees of perfection. If the reasoning so far is correct, then perfection is strictly increasing in our lineage of gods.

The ordinal hierarchy of gods

Each god in our lineage self-reproduces using some athletic theogenic algorithm f. Any such algorithm always searches for greater perfection; but any such search is some version of Leibnizian Search. Thus f is the Leibnizian Search for gods. Since gods are hardware objects, f is more precisely referred to as the *Leibnizian Search for Hardware* (LHARD). To define it, let *Alpha* be any self-reproducing god. As part of its self-knowledge, Alpha has a description of its nature. So LHARD has three steps: First, Alpha makes a set of more perfect versions of its self-description.[22] These describe its *potential* offspring. They are variants of Alpha that it *can* actualize. Second, Alpha sorts these descriptions into ranks based on their perfections. This sort necessarily converges to some top rank, which contains the descriptions of the *best* potential offspring of Alpha.[23] Clearly, Alpha ought to actualize each of its best potential offspring. Third, since Alpha does what it ought to do, Alpha actualizes every one of its best potential offspring.

Since our lineage has the form $\langle g, f^1(g), f^2(g), \ldots f^n(g)\rangle$, and since f is LHARD, every god in our lineage up to our god runs LHARD. And every god in our lineage passes LHARD down to its offspring. Thus our god *can* run LHARD. But does it? The *Argument for Divine Success* aims to show that god–god productivity does not fail: (1) Say a god is *proactive* iff it has the ability and the desire to run LHARD. If a proactive god does not run LHARD, then it must be blocked by some concrete or abstract obstruction. (2) Consider concrete obstructions. Since gods are the deepest concrete things, there are no deeper concrete things that could interfere with their desires. Since gods are solitary, they do not interfere with each other. It makes no sense to say that any god interferes with itself. So no concrete obstructions stop any proactive god from running LHARD. (3) Consider abstract obstructions. Mathematics provides transfinitely endless ways to increase computational power and intelligence (Steinhart ([2003])). Since gods are computational, it follows that no abstract obstructions stop any proactive god from running LHARD. (4) Therefore, any god that has the ability and desire to run LHARD does run it.

Since our god is optimal, it has maximal benevolence towards itself. Such self-optimality plausibly entails a desire to self-reproduce. Granted this reasoning, our god desires to self-reproduce. It therefore has the desire to run LHARD. Since our god has the ability and desire to run LHARD, the Argument from Divine Success entails that our god does run LHARD. Thus our god makes offspring. But now all the reasoning about LHARD applies to them too. Consequently, for any god, if that god is produced by LHARD, then it inherits LHARD. And if it inherits LHARD, then it runs it. Starting from our simple god G(0), LHARD runs endlessly, making an infinitely ramified tree of ever more perfect gods. To formalize this *great tree*, it is useful to say that a *way to improve* a god is a way to change it into

one of its best potential offspring and that an *improvement* of any god is a way to improve it. Now two rules define the great tree. The *initial rule* says that the great tree contains our simple god. The *successor rule* says that for every god in the great tree, for every way to improve it, there exists some god in the great tree that is improved in that way. Although the term *divine* has been used loosely so far, the time has come to formalize it: say a mind is *divine* iff it runs LHARD. Every god in the great tree is divine.

Since our simple god G(0) is simple, and since any god in the great tree so far has gone through only finitely many iterations of self-improvement, it may appear that every god in that tree is only finitely complex. Perhaps our god is only finitely complex. Although ordinal polytheists take no position on whether our god is finitely or infinitely complex, they agree with tradition that *some* gods are infinitely complex. The great tree already contains progressions of gods with the length of the natural number line. Since these progressions are infinitely long, it seems logical to extend the concept of divine creative power to the transfinite by asserting that if any extension of any divine progression to any limit is consistent, then it happens. For any infinite progression of ever more perfect gods, there exists a *limit god* that is more perfect than every god in that progression.[24] Adding this limit rule allows divine progressions to run into the transfinite. Since the transmission of LHARD is preserved from any god to its successor, and since the definition of limit gods permits such transmission in the limit, it is plausible to say that it is preserved from any infinite progression to its limit. Granted that such transmission is continuous at limits, ordinal polytheists affirm that every limit god runs LHARD.

Three rules define the *Divine Hierarchy* of gods. They are based on the *Long Line* of ordinals defined in some standard set theory (e.g., ZFC plus axioms for all consistently definable large cardinals – see Drake (1974)). The *initial rule* says that for the initial ordinal 0 on the Long Line, there exists an initial generation D(0) of unproduced gods. This contains at least G(0). Every god in D(0) is an ultimate necessarily existing divine mind. Ordinal polytheists need not decide whether there are many simple gods in D(0). The *successor rule* says that for every successor ordinal $n + 1$ on the Long Line, there exists a successor generation D$(n + 1)$ that contains every improvement of every god in D(n). The *limit rule* says that for any limit ordinal L on the Long Line, there exists a limit generation D(L) that contains every limit of every progression of gods through ranks less than L. Every god in the Divine Hierarchy is a necessarily existing divine mind.

These rules entail that there are infinitely many gods, some of which are infinitely perfect. Our god may be infinitely perfect. However, just as every ordinal is surpassed by bigger ordinals, so every rank of gods is surpassed by ranks of more perfect gods (Steinhart (2012), sec. 5). Ordinal polytheism thus avoids the paradoxes of omniscience (Grim (1988)) and omnipotence (Cowan (1965)). If any

god knows each truth in some set, then some more intelligent god knows each truth in some bigger set; if any god can do some hard act, then some more powerful god can do some harder act.

The divine generation of universes

Our god is extremely complex. Ordinal polytheism says that the complexity of our god is explained by the iteration of LHARD. However, it may be objected that ordinal polytheism has not provided a sufficiently detailed explanation for that divine complexity. After all, it has been previously argued that our god runs an extremely complex Leibnizian Search for the best of all cosmic descriptions. By contrast with the Leibnizian Search for Hardware, this is a *Leibnizian Search for Software* (it is the search for the programme that our god runs on its shell, and whose execution generates our universe). This search can therefore be called LSOFT. Much of the complexity of our god lies in the complexity of its version of LSOFT. Therefore, any explanation for the complexity of our god must include some detailed explanation for the complexity of its version of LSOFT. But ordinal polytheism has not yet explained the complexity of the version of LSOFT run by our god.

To explain the complexity of our version of LSOFT, ordinal polytheists offer an *Argument for the Accumulation of Search Skill.* Here it is: (1) Our god is at the end of our lineage. (2) Each later god in our lineage is produced by the iteration of LHARD starting from our simple initial god. (3) Our god runs an extremely complex version of LSOFT. (4) Say our lineage is *robust* iff our simple initial god runs some simple version of LSOFT and each iteration of LHARD on the gods in our lineage always produces offspring gods that run more complex versions of LSOFT. (5) If the Principles of Ascent and Design Accumulation are correct, then the most likely explanation for the fact that our god runs its version of LSOFT is that our lineage is robust. (6) But the most likely explanation is the best. (7) So, by inference to the best explanation, our lineage is robust. This means that every god in our lineage runs some version of LSOFT and that the complexity of that version increases from earlier to later gods in our lineage. Our version of LSOFT lies at the end of a long cumulative walk in which the complexity of versions of LSOFT is increasing.

According to the Argument for the Accumulation of Search Skill, our lineage is robust; but this entails that any lineage that branches off from our lineage is robust. More generally, it entails that every lineage that starts with our simple god, and which develops via the iteration of LHARD, is robust. And since simple gods must be extremely similar, ordinal polytheists infer that it is plausible that every lineage that starts with any simple god, and which develops via the iteration of LHARD, is also robust. But this means that every lineage in the Divine Hierarchy is robust. Several consequences follow immediately: first,

every god in the Divine Hierarchy runs some version of LSOFT. Second, the complexity of that version increases along the offspring relation from lower to higher ranks of gods. And third, every god divides functionally into its kernel and its shell. On its kernel, it runs LHARD and LSOFT; on its shell, it runs the programme discovered by its version of LSOFT. Say that a *pattern* is any programme that runs on the shell of some god. Since every god runs some version of LSOFT on its kernel, and since LSOFT always discovers exactly one pattern for that god to run on its shell, every god runs exactly one pattern on its shell.

Since the way that every god runs its pattern is exactly analogous to the way that our god runs our pattern, ordinal polytheists extend their analysis of physicality from our god to all gods. Just as any object that supervenes on the shell of our god is a physical thing, so any object that supervenes on the shell of any god is a physical thing. On any god, some class of physical things supervenes. And the class of all physical things is the union of the classes of physical things at all gods. But the class of all physical things divides into disjoint universes. Just as any two physical things are in our universe iff they supervene on the shell of our god, so any two physical things are in the same universe iff they supervene on the same shell of the same god. By running its pattern, every god causes every physical thing in its universe to exist. However, this divine causality is not physical; it is deeper than physical. Every god thus serves as the necessarily existing hardware ground of some system of contingent software objects. Every god in the Divine Hierarchy generates exactly one universe. By running (its version of) LSOFT, every god designs its universe; by running the pattern discovered by LSOFT, every god creates its universe.

Our god, like any god, runs exactly one pattern on its shell. By running its pattern, it generates our universe. Since our universe is extremely complex, our pattern has some very high degree of complexity. But the scientific analysis of complexity says that any complex thing lies at the end of some cumulative series. Thus our pattern lies at the end of some cumulative series of patterns. On the one hand, our god builds this cumulative series of patterns entirely within itself. This requires massive search. On the other hand, our god merely extends the work of its ancestors. Of course, since its ancestors already support a cumulative series of increasingly complex patterns, merely extending their work is by far the most economical way for our god to make our highly complex pattern. For the sake of economy, ordinal polytheists say the best explanation for the complexity of our pattern is that our god merely extends the work of its ancestors. This extension involves two steps: (1) our god inherits some *seed* pattern from its ancestors; (2) our god uses its inherited seed pattern as the starting point for its own local search for our pattern.

On the basis of their earlier reasoning about the perfections of all gods, ordinal polytheists say that this logic generalizes: every god inherits some pattern from its

ancestors and uses it to derive its own pattern.[25] Consider how gods inherit their patterns. Any god in any progression is either an initial god, a successor god, or a limit god. Any initial god inherits the empty pattern. Any successor god inherits the pattern of its predecessor. Any limit god inherits the limit of the patterns of its ancestors.[26] Now consider how gods derive their own patterns from their inherited patterns. Since every god is optimal, it does what it ought to do. And it ought to do the best that it can do. Say that pattern *p is better than* pattern *q* iff the universe generated by running *p* is better than the universe generated by running *q*.[27] And one universe *x* is better than another universe *y* iff the *entire change* from *y* to *x* is morally justified. Since every god does what it ought to do, every god runs the best pattern it can run. This reasoning leads to a more precise definition of LSOFT.

More precisely, the operation of LSOFT at any god involves four steps: First, the god derives a set of local variants of its inherited seed pattern. Any simple pattern is a local variant of the empty pattern. Second, it performs an axiological sort that stratifies those variants into ranks. Every axiological sort necessarily converges to some top rank. These are the patterns the god can run (the ones whose executions are consistent with its perfection). Since every god is more perfect than its ancestors, its top-ranked patterns are all better than its seed. Third, the god selects one of its top-ranked patterns for execution. If there are many such patterns, the god *freely* chooses one. Each god therefore freely chooses exactly one top-ranked pattern for execution. This is the best pattern it can run (its best pattern). Since every god is optimal, it runs its best pattern. By running its best pattern, the god generates all and only those *thoughts which for it are worth thinking* (Leslie (2001), 23–39). All the things in its universe supervene on those thoughts. Since its best pattern is better than its inherited pattern, its universe is better than any of its ancestral universes.

Since every god generates exactly one universe, the class of universes inherits much of the structure of the class of gods (of the Divine Hierarchy). The partitioning of gods into ranks induces a parallel partitioning of universes into ranks. The gods in D(n) generate all and only the universes in R(n). The offspring relation among gods induces a parallel relation among universes. Of course, universes do not cause other universes to exist. But no harm is done by analogically extending the ancestor and descendant relations to universes. Say that U(x) is the universe designed and created by god *x*. Thus god *x* is a descendant of god *y* iff U(x) is a descendant of U(y). The improvement relation among gods induces a parallel relation among universes. If god *x* is an improvement of god *y*, then U(x) is a better version of U(y). Since descent for gods entails improvement, it follows that if U(x) is a descendant of U(y), then U(x) is a better version of U(y). Thus if *n* is greater than *m*, every universe in R(n) is better than every universe in R(m).[28] The *Axiarchic Class* of universes is the union of the R(n) for every ordinal *n* on the Long Line.

Ordinal polytheism and modal logic

Monotheists say that our god exists necessarily. Divine necessity is justified by arguments like Aquinas's Third Way (*Summa Theologica*, Part 1, Q. 2, Art. 3) and by the modal ontological arguments (e.g. Leftow (1988)). Monotheists also say that our god is concrete. They will thus endorse the following argument: (1) Our god necessarily exists. (2) Our god is concrete. Therefore (3) our god is a necessarily existing concrete being.

Ordinal polytheists agree with monotheists that our god is a necessarily existing concrete being. The analysis of necessity falls within the metaphysics of modality. One approach to modality is Leibnizian. And some have considered the thesis that the Leibnizian approach allows our god to have counterparts (Wilson (1979); Blumenfeld (1982)). Another approach to modality is the modal realism of David Lewis (1968; 1986).[29] Divine necessity and Lewisian modal realism seem to imply that our god has counterparts (see Le Poidevin (1996), 27–31; Sheehy (2006); for a contrary opinion, see Cameron (2009)).

For ordinal polytheists, the metaphysics of modality is mainly Lewisian. Over every god, there exists one *world*.[30] Worlds are concrete; they do not overlap; worlds contain things; some of those things are counterparts. More formally, the *manifestation* of any god is the set containing all and only the god, its parts, and all things that supervene on those parts. Of course, worlds are not synonymous with universes; every universe is a proper part of some world. The world over any god is $(M, P_1, \ldots P_m, R_1, \ldots R_n)$ where M is the manifestation of the god, each P_i is a property over M, and each R_i is a relation over M.

The class of gods determines a class of worlds. The class of worlds satisfies the postulates of Lewisian counterpart theory and serves a domain for modal quantification. The analysis of the *de dicto* modes goes exactly as expected. To put it informally, for any proposition P, it is *necessary* that P iff P is true at every world; it is *possible* that P iff P is true at some world. The analysis of the *de re* modes uses counterpart theory. To put it informally, for any property *p*, for any thing *x, x* is *essentially p* iff *x* is *p* and every counterpart of *x* is *p*; and *x* is *accidentally p* iff *x* is *p* but some counterpart of *x* is not *p*.

Every god is a counterpart of every other god (and only of every other god). Since every god has a counterpart at every world, every god exists necessarily. The *divine essence* is the conjunction of all and only the essential properties of gods. For ordinal polytheism, the divine essence includes at least these features: every god is a pure mind; every god has parts; every god has a kernel and a shell; every god does the best it can do; every god has some positive degree of perfection; every god runs LHARD to design and create some offspring gods; every god runs LSOFT to design and create its universe.[31]

Although theology focuses on gods, the physical sciences focus on universes. More precisely, the *universe* over any god is $(S, P_1, \ldots P_m, R_1, \ldots R_n)$ where S is the

set of physical things that supervene on the shell of that god, each P_i is a property over S, and each R_i is a relation over S. Each universe is a proper substructure of the world of its god. Hence the class of worlds determines a class of universes. This class is the domain for a restricted type of modal quantification. The restriction is *physical* (or, perhaps better, *cosmological*). Thus it is cosmologically necessary that P iff P is true at every universe; it is cosmologically possible that P iff P is true at some universe. For the *de re* modalities, the counterpart relation is restricted to universes and things in them.

There are two main differences between the *Lewisian Class* of universes (Lewis (1986), sec. 1.8) and the Axiarchic Class of universes (the union of the R(n) for every ordinal n). While the definition of the Lewisian Class does not involve stages, the definition of the Axiarchic Class does involve stages. While the definition of the Lewisian Class does not involve any axiological constraints, the definition of the Axiarchic Class involves axiological constraints that eliminate merely combinatorial options at every stage. These two differences ensure that the Axiarchic Class is a small proper subclass of the Lewisian Class. While the Lewisian Class is closed combinatorially (Lewis (1986), sec. 1.8), the Axiarchic Class is not.

Improvement in the art of world-making

All the gods in the Divine Hierarchy are surpassably perfect. Alas, Kane (1984, 344) portrays such surpassable gods as *defective* (e.g. as doing evil). Ordinal polytheists reply that such negativity is not consistent with their concept of variable perfection. Every god has some non-zero degree of perfection. But every such degree is some degree of pure positivity, however small: no god has any malevolence, no god affirms any falsehoods. All gods do some good; they all know some truths; they all have some power.

The less perfect gods are just *simpler* gods that actualize *simpler* universes. Henle (1961, 102–103) describes three gods. The first god is *Nec*. Nec is not able to actualize universes containing motion. The second god is *NEc*. NEc can actualize universes containing only uniform rectilinear motion. The third is *NEC*. NEC can actualize universes containing acceleration. The universes actualized by these simple gods are merely simple universes – universes with less physical richness. It may be helpful to go into more detail.

Suppose Alpha is a god of low perfection. Alpha knows only how to make two-dimensional cellular automata (CAs). The complexities of our physics are beyond its ken. And, relative to these 2D CAs, Alpha has an appropriate sense of goodness. Surveying these universes, Alpha is able to select one that satisfies an appropriate concept of the best. Alpha creates that universe. Surely Alpha has not believed any falsehoods or done anything crazy or wrong. On the contrary, Alpha did the best that Alpha could do.

Suppose Beta is some god of somewhat greater perfection than Alpha. Beta has greater intelligence than Alpha. Hence Beta is able to conceive of universes that lie (in terms of complexity) between 2D CAs and our universe. As it surveys its understandable universes, Beta reasonably decides that a universe populated only by counterparts of our bacteria is the best that it can make – hence it makes that universe. Although Beta has only some small positivity, Beta is neither incompetent nor malicious nor feeble. And the fact that Beta is more perfect than Alpha does not render Alpha negative in any way.

Along any lineage in the Divine Hierarchy, the gods are like Hume's shipwrights, and there is a 'slow, but continued improvement . . . in the art of worldmaking' (Hume ([1779/1990](#)), 77). For some ordinal k, our god is the k-th god in some lineage in the Divine Hierarchy. It sits in the k-th rank D(k). Our god does the best it can do in making our universe; yet our god can be improved. And if it were improved, then it would do better. If our god were more perfect, then it would create a better universe. To say that our god *can be* improved means that our god has counterparts who *are* improved. These are the gods in higher ranks. It is a morally redeeming feature of ordinal polytheism that every god can say of itself: although I do the best I can, if I were more perfect, then I would do better.

Since every god in the Divine Hierarchy is surpassed, none of those gods are vulnerable to any *logical argument from evil.* However, it may be that those gods are vulnerable to *evidential arguments from evil.* Perhaps the evil in our universe makes it improbable that it was made even by our surpassable god. Against our god, the evidential argument from evil says that it is probable that any god that can make a universe as physically complex as ours can also make universes that are morally better than ours. Our universe thus has no moral justification. And perhaps the same holds at other universes. Ordinal polytheists, of course, will want to try to find ways to block such evidential arguments from evil.

Conclusion

Although ordinal polytheism is unorthodox, it nevertheless has plenty of theologically attractive features to advertise. First, and probably foremost, ordinal polytheism is a type of purely natural theology. It begins with manifest evidence, which it interprets using the ideas and techniques of the formal sciences. Since it has no prior allegiance to any revelation, it can follow the arguments wherever they lead. It aims to be a theology without any mystery – it is entirely open to rational criticism and correction. Second, by thinking of universes as software processes running on divine computational substrates, ordinal polytheism avoids old problems with the god–universe relation. Third, thinking of gods in computational terms enables ordinal polytheists to make precise algorithmic models of their gods. Ordinal polytheists can learn from both the successes *and the failures* of these models. Thus ordinal polytheism can hope to make *theological*

progress much as other forms of human knowledge make scientific progress. Fourth, for ordinal polytheists, concepts like *simplicity* and *infinity* take their meanings from the formal sciences. Ordinal polytheists are therefore able to offer scientifically meaningful analyses of divine simplicity and infinity. Fifth, since its gods are all surpassable, ordinal polytheism avoids problems with the vacuity or coherence of an unsurpassable god. Thus ordinal polytheism avoids at least the logical problem of evil. Every god does the best it can do; but it cannot be faulted for not having done better; it would do better if it could. To be sure, ordinal polytheism, like any philosophical position, will generate problems of its own. Ordinal polytheists welcome philosophical criticisms and refinements of their theological positions.

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Notes

1. Ordinal polytheism is a kind of purely natural theology. Its gods do not inherit any of the strictly revealed features of the Abrahamic deities. It is immune to the doctrinal arguments against the Christian God (see Martin & Monnier (2003), part 3).

2. Modern mathematicians use the transfinite endlessness of the ordinal number line and the transfinite endlessness of the iterative hierarchy of sets to define transfinitely endless hierarchies of ever greater languages, computers, games, and strategies.

3. Accidentally ordered causes may be infinitely regressive; however, series of essentially ordered causes must be well-founded (Wengert (1971); Harrison (1974)). For ordinal polytheists, dependency is analogous to computational supervenience: things depend on things as virtual machines supervene on deeper machines. But stacks of virtual machines are necessarily well-founded; they are essentially ordered.

4. The class of Organic Design Arguments includes the Fifth Way in Aquinas (*Summa Theologica*, Part 1, Q. 2, Art. 3); and Paley's (1830) watchmaker argument.

5. The class of Cosmic Design Arguments includes at least the Stoic design argument in Cicero (*De Natura Deorum*, bk. 2); the cosmic engineers argument in Hume (1779/1990, 53); and all the fine tuning arguments (Leslie (1989)).

6. For infinite computers more powerful than Turing machines, see Copeland (1998), Hamkins (2002), Koepke (2006), Koepke & Siders (2008).

7. There are many ways to define complexity in terms of information. Note that algorithmic incompressibility (*à la* Kolmogorov) measures randomness but *not* complexity. Ordinal polytheists are especially interested in those informational definitions of complexity that involve *depth*. These include *thermodynamic depth* (Lloyd & Pagels (1988)); *computational depth* (Antunes et al., (2006), (2009)) and *logical depth* (Bennett (1988), (1990)). The definition of complexity in Dawkins (1996, 6–9) seems equivalent to depth.

8. The term 'intricate', once introduced, will hardly ever be needed. It will be clear from the context whether an object is one of the simplest of its type or whether it is intricate.

9. The complexity of a theory is its number of quantifiers (Kyburg (1961)); or its number of quantifier alternations (which determine its rank in the Kleene–Mostowski hierarchy); or the way of counting quantifiers in Krynicki & Szczerba (1990); or so on.

10. Hume says 'the cause ought only to be proportioned to the effect' (Hume (1779/1990), 76). Since our god is the cause of our universe, it is thus proportional.

11. Leibniz portrays the divine memory as containing uncountably infinitely many bits (Leibniz (1710/1996), secs. 414–417) and the divine thought as performing uncountably infinitely many operations on them (*ibid*., sec. 225). Royce (1899/1927) says that the Absolute (that is, our god) contains every series $\langle n, f(n), f(f(n)), \ldots \rangle$ where n is in the set of natural numbers N and f is any map from N into N; hence his Absolute is uncountably complex.

12. It seems very easy to define gods far less complex than our god. Consider the least complex god that searches through the class of two-dimensional totalistic cellular automata to find those that support universal Turing computation. Surely such a god is possible. And, given the obvious complexity of current physics, it seems hard to argue that our god is less complex. If this reasoning is accepted, then our god is not simple; it is intricate.

13. Against the complexity of the divine mind, Craig attempts to argue that while the divine thoughts are complex, the divine mind itself is simple (Craig (2008), 171–172). Yet since thoughts are at least states of minds, a mind with complex thoughts is a complex mind. And it is hard to understand how a thing with complex states can fail to have parts.

14. Following Bower (1988), the complexity of a genotype is proportional to the number of distinct cell types in its phenotype. Adami et al. (2000) define the complexity of an organism directly in terms of genetic information. Nehaniv & Rhodes (2000) provide an insightful mathematical analysis of biological complexity.

15. For the mathematics of complexity, ordinal polytheists turn mainly to accounts based on *depth*. Bennett proves that logically deep objects obey 'a slow-growth law: deep objects cannot be quickly produced from shallow ones by any deterministic process, nor with much probability by a probabilistic process, but can be produced slowly' (Bennett (1988), 227). Antunes et al. (2006; 2009) prove that analogues of slow-growth laws hold for computational depth. All these slow-growth laws support the Principle of Ascent.

16. Perhaps it should only be said that every possible *concrete* thing is at the end of some cumulative walk. However, the iterative conception of sets at least suggests that the Principle of Ascent holds even for *abstract* objects.

17. Dennett's Principle of Accumulation of Design says:

> since each new designed thing that appears must have a large design investment in its etiology somewhere, the cheapest hypothesis will always be that the design is largely copied from earlier designs, which are copied from earlier designs, and so forth. (Dennett (1995), 72)

18. Dawkins says that it is almost certain that our universe is not designed. However, he admits that there is some small probability that it is designed. He therefore develops several rules for this design. If our universe is designed, then (1) its designer (our god) is complex (Dawkins (2008), 178–179); (2) our god 'must be the end product of some kind of cumulative escalator' (*ibid.*, 186); (3) the series of ever more complex gods starts with some first cause (*ibid.*, 185); and (4) '[t]he first cause that we seek must have been the simple basis for a self-bootstrapping crane' (*ibid.*, 185).

19. Divine self-reproduction is causal but deeper than physical. It is ur-physical. Hence gods participate in *ur-causal* relations with other gods. And divine self-reproduction determines an ur-physical temporal ordering among gods. Hence gods participate in *ur-temporal* relations with other gods. Ancestors are ur-temporally earlier than their descendants. Within any god, some of the parts of its shell are *ur-spatially* related. However, since none of the arguments about gods suggests that they participate in any ur-spatial relations with each other, ordinal polytheists say that they do not participate in such relations.

20. Aquinas writes, in his Third Way, that 'every necessary thing either has its necessity caused by another, or not' (*Summa Theologica*, Part 1, Q. 2, Art. 3). And many writers allow complex necessary beings to depend on simpler necessary beings, as, for instance, complex pure sets depend on the simpler pure sets which are their members.

21. Ordinal polytheists are therefore committed to defending the view that supererogation is not possible for their gods. They may defend this by saying that any will which is not obligated to do the best it can do is less than maximally benevolent. There are interesting issues here which lie outside the scope of the present discussion.

22. It has been argued that intelligent computers will apply their minds to their own designs in order to make superior versions of themselves (Kurzweil (2005); Schmidhuber (2007); Chalmers (2010)). Since such computers get better at making themselves better (at making their offspring more powerful and intelligent), it is said that they are engaged in *recursive self-improvement.* Adopting this idea, ordinal polytheists say that any god that runs LHARD is also engaged in recursive self-improvement.

23. Since failure to produce a top rank leaves any god in a state of paralysis, every god necessarily performs an axiological sort that is guaranteed to converge onto a top rank. When any such sort produces an infinitely axiologically ascending hierarchy of ranks, the god forms a top limit rank that is the axiological supremum of that hierarchy.

24. Along any series of gods, the *lineage to n* is the fusion of all gods with indices less than n. A lineage x is *transitive* iff it is well ordered and every lesser god in x contributes to the production of every greater god in x. Since productivity is cumulative, every lineage is transitive. And again, since productivity is cumulative, it is consistent with divine solitude to say that for any finite or infinite ordinal n, and for any x, if x is any transitive lineage to n, then x produces the n-th god. Clearly, if L is some limit ordinal, then the L-th god lacks a predecessor, and so cannot be produced by its predecessor. But the design and creation activities of all lesser gods contribute to its existence. Hence for any limit ordinal L, the L-th god is produced by the lineage to L of gods.

25. The inherited pattern may be highly decorated. Moral considerations suggest that every inherited pattern is decorated with the details of every morally valuable thing that was generated by the

execution of that pattern. Since the earlier gods ran the earlier patterns, they know all the details of all those morally valuable things.

26. Any god x runs a pattern S(x). For any ordinal k, for any progression of gods G, the series of patterns up to k is the series of S(k) for all gods with indices less than k. For any limit ordinal L, the *limit of a series patterns up to* L is the simplest pattern p such that all the complexity in p is explained by contributions from the patterns in the series up to L. For any limit ordinal L, for any progression of gods G, any limit god G(L) inherits the limit of the series of patterns up to L. Thus limit gods inherit limit patterns.

27. Since universes contain things with moral value, the analysis of the better-than relation among universes is likely to involve counterparts. One analysis of that relation based on counterparts looks like this: for non-empty universes x and y, say y *is a better version of x* iff (1) every valuable thing in x has a counterpart in y; (2) distinct valuable things in x have distinct counterparts in y; (3) no valuable thing in x has a worse counterpart in y; and (4) at least one valuable thing in x has a better counterpart in y. This analysis can serve as the basis for a counterpart-theoretic version of the pareschatology of John Hick ((1976), chs. 15, 20; see Steinhart (2008)). Ordinal polytheism thus has at least one soteriology.

28. For every god, for every way to improve that god, there is some god that is improved in that way. Every god makes a better universe. Now, for any n, if there is any better version of a universe in R(n) that is missing in R($n+1$), then there was some way to make an improvement of a god in D(n) that is missing from D($n+1$). But no such gods are missing from any D($n+1$). It follows that, for every universe in any R(n), for every way to make a better version of that universe, there is some universe in R($n+1$) that is made better in that way. If this reasoning is correct, then the system of universes is *axiologically complete* at successor ordinals. And this logic is easily extended to R(L) for limit ordinals L. But this entails that the entire system of universes is axiologically complete.

29. Ordinal polytheists reject Lewisian polytheism (Lewis (1983), xi; note 1).

30. Monotheists say x is a god only if x is the source of the being of all other things (see Le Poidevin (1996), 30). Ordinal polytheists say x is a god *at some world* only if x is the source of the being of all the other things *at that world* (Sheehy (2006), 319).

31. If any god finds more than one top-rank pattern, then it freely chooses to execute exactly one. This freedom gets the expected counterpart-theoretic analysis (see De Sousa (1974), 449, fn. 7; Wilson (1979), 723–724). A god *freely* chooses one top-rank pattern as its best pattern iff it could do otherwise; it could do otherwise iff it has counterparts who do otherwise. They are its siblings (its rankmates).